Network effects and Cournot competition

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Main Points

- Consider 2 models: Industry-wide and firm-specific networks
- Stress differences between network and regular industries
- Formalize industry viability (network industry take off) and show that it is enhanced with more firms
- Study effects of market structure on performance
- Revise notion of free entry equilibrium to get a unique solution
- Closed-form examples with multiple equilibria
- Policy insights: Antitrust (allow some inter-firm coordination)
- Use lattice-theoretic methods, deal with multiple equilibria

Related Studies

- Veblen (1899)
- Leibenstein (QJE 1950), Rohlffs (Bell J. 1973), Economides
- Rohlffs (2001): book on history of network industries
- Katz and Shapiro (*AER*, 1985)

"Network externalities, competition and compatibility"

Key issues with past work

- Little attention to generality and robustness of results
- No formal modeling of industry viability (or take off) issue, though this is often discussed
- Little attention to free entry in network industries
- Distinguish pure and mixed network goods

Model, Equ Concept and Assumptions

- n identical firms sell a homogeneous and fully compatible good
- The profit function of a firm is (S is expected network size)

$$\pi(x, y, S) = x[P(x + y, S) - c]$$

or

$$\widetilde{\pi}(Z, y, S) = (Z - y)[P(Z, S) - c]$$

- A rational expectations Cournot equilibrium (RECE) is a vector $(x_1^*, x_2^*, \ldots, x_n^*, S^*)$ such that

(i) $x_i^* \in \arg\max_x \left[xP\left(x + \sum_{j \neq i} x_j^*, S^* \right) - cx \right]$

(ii) $S^* = \sum_{i=1}^n x_i^*$

Some thoughts on RECE

- Hybrid concept: firms are strategic in output choice but network size-taking
- RECE is suitable when firms lack commitment power (Katz-Shapiro, 85)
- RECE are equivalent to SPE of two-stage game with market maker acting in stage 1
- RECE reflects co-opetition: Firms are partners in building a common network but competitors in serving it

Assumptions

A1 $P_1(Z,S) < 0$ and $P_2(Z,S) > 0$

A2 $\Delta_2(Z,S) = P(Z,S)\,P_{12}(Z,S) - P_1(Z,S)\,P_2(Z,S) > 0$

Key implications

$\widetilde{\pi}(Z,y,S) = (Z-y)\,[P(Z,S)-c]$

- A2 $\iff$ $[\log P(Z,S)]_{ZS} > 0 \iff [\log \widetilde{\pi}(Z,y,S)]_{ZS} > 0$
- A symm. Cournot equ. exists for each S.

Results

MODEL I : Industry-wide network

- Let $Z_n(S)$ be Cournot equ. total output given network size S.
- RECE are the fixed points of $Z_n(S)$.
- Key property: $Z_n(S)$ is increasing in S

Theorem

There is a symmetric equilibrium and no asymmetric equilibria.

Main idea. By Tarski FPT, a fixed point exists, and is a RECE.

- For pure network goods, $P(Z,0) = 0$
  $\implies Z_n(0) = 0$ is a RECE
  Zero expectations are self-fulfilling
- More generally, low stand-alone value $\implies$ 0 is a trivial RECE

Lemma

For any n, $Z_n(0) = 0$ is a RECE iff $P(0,0) \leq c$.
If $Z_n = 0$ is a RECE for some n, it remains one for all n.

- Fixed point approach is inadequate.
  Need to show existence of non-trivial RECE.

Existence of non-trivial RECE

Theorem (NTE)

A non-trivial equilibrium exists if at least one condition holds

(i) $Z_n = 0$ *is not a RECE;*

(ii) $Z_n = 0$ *is a RECE and* $n > \dfrac{-P_1(0,0)}{P_1(0,0) + P_2(0,0)}$*; or*

(iii) $Z_n = 0$*, and for some* S *and all* $Z \leq S$*,*

$$P(Z,S) + \frac{Z}{n} P_1(Z,S) \geq c$$

Industry Viability

Define expectations/network size dynamics, starting from S_0,

$$S_k = Z_n\left(S_{k-1}\right), k \geq 1.$$

Definition

An industry is said to be

(i) uniformly viable if $\lim S_k > 0$ from any $S_0 > 0$

(ii) conditionally viable if $\lim S_k > 0$ from any $S_0 > 0$ high enough

(iii) nonviable if $\lim S_k = 0$ from any $S_0 \geq 0$.

Note. Simplifying assumption in past work $P\left(Z, S\right) = p(Z) + f\left(S\right)$

- *Key implication*: uniform viability

Market structure/technology and viability

Theorem

$Z_n(.)$ shifts up as
(i) n increases, and/or
(ii) α increases.

Hence, industry viability always increases with

(i) more competing firms and/or

(ii) technological progress (or process R&D).

MODEL II : firm-specific network

- Inverse demand is $P(Z,s)$, $s =$ firm-specific network size
- Let $q_n(s) =$ Cournot equ. per-firm output given s
  Hence $q_n(s) = Z_n(s)/n$
- FECE are fixed points of $q_n(s)$.
- Similar conditions for existence

Viability

- If $P(Z,s)$ is log-concave in Z, $q_n(s)$ decreases in n.
- Viability decreases in n (monopoly is best)!
- Viability is better in Model I.

Case Study: The fax industry (Rohlffs, 2001).

- First failed launch mid-1800's in Lyon, France
  Firm-specific networks and bad technology
- Successful launch in 1980's : Single network
  after gov't mandated interconnection
- Rohlffs (2001) : This is a general pattern
  Gov't intervention can help viability in many ways:
  create a single network, coordinate entry, increase S_0...

Example

$$P\left(Z,S\right)=\exp\left(-\frac{2Z}{\exp(1-1/S)}\right)$$

Usual 3-equ. configuration with analytical solutions

- $n=1$: industry not viable
- $n=2$: borderline case (tangency point as RECE)
- $n\geq 3$: industry conditionally viable
- π_n is not decreasing in n here.

In fact, $\pi_1=0$: A monopolist would want some competitors !

Comparison of the 2 models

- Outputs. $x_n^I \geq x_n^{II}$ (or $Z_n^I \geq Z_n^{II}$)
- The (endogenous) inverse demands satisfy $P(\cdot, \bar{z}_n^C) > P(\cdot, \bar{x}_n^I)$
- Price. $P_n^I \geq P_n^{II}$ if $-P_1 P_2 + z(P_1 P_{12} - P_2 P_{11}) \geq 0$
- $CS_n^I \geq CS_n^{II}$ if $P_n^I \leq P_n^{II}$ or $P_{12} \leq 0$
- Profit. $\pi_n^I \geq \pi_n^{II}$ if
  $-P_1(2P_2 + xP_{12}) + z(P_1 P_{12} - P_2 P_{11}) \geq 0.$

  Ex. $P(Z, S) = Se^{-Z}$. Then $\pi_n^I = ne^{-n} < \pi_n^{II} = e^{-n}$
- $W_n^I \geq W_n^{II}$
  (large gap if Model I is viable and Model II is not)

Free-entry Equilibrium (FEE) in Model I

- Consider standard two-stage game (entry cost K):

Stage 1. each firm decides: Enter or not

Stage 2. RECE amongst entrants.

- FEE # of firms n^e is defined by $\pi_{n^e} \geq K$ and $\pi_{n^e+1} < K$
- In standard oligopoly, π_n is decreasing in $n \implies$ FEE is unique.
- Since π_{n^e} is not decreasing in n, multiple FEE are possible (i.e., $\pi_{n^e} = K$ has multiple solutions)
- If monopoly non-viable $\implies n^e = 0$ is a FEE (no industry!)

For a unique FEE outcome, we need a new equilibrium concept.

Two possible solutions:

- An equilibrium refinement:
  Use Coalition-Proof Nash Equ. in entry decision.

Then FEE yields the maximal # of firms sustainable in industry as unique outcome (this is also practitioner's definition).

- A sequential entry process in stage 1:
  Change Stage 1 to sequential entry

  Then maximal # of firms sustainable in industry is unique outcome

On the theoretical scope of RECE

How broad is the class of $Z_n(\cdot)$'s that can be rationalized?

Answer: Any function $h(S)$ with $h' > 0$

Consider the inverse demand

$$P(Z, S) = \exp[-nZ/h(S)].$$

Given S, $\exists$ a (dominant-strategy) Cournot equ. $x^* = h(S)/n$, so

$$Z^*(S) = h(S)$$

In particular, if one takes $h(S) = S$, any feasible outcome is a RECE !

Model with commitment

- Here, firms can commit to output levels and influence consumers' expectations of S (Katz-Shapiro 85)
- The model then boils down to standard Cournot with inverse demand $P(Z, Z)$.
- This demand is downward-sloping iff output effect outweighs network effect, i.e

$$P_1(Z, Z) + P_2(Z, Z) < 0$$

The profit function of a firm is

$$\pi(x, y, S) = xP(x + y, x + y) - cx$$

After change of variable

$$\widetilde{\pi}(Z, y, S) = (Z - y)[P(Z, Z) - c]$$

Key Results

- Firms act as if $P(Z, Z)$ were downward-sloping (with flat spots), i.e as if
$$\widehat{P}(Z) = \sup_{a \geq Z} P(a, a)$$
$\widehat{P}(Z)$ is the smallest decreasing function that majorizes $P(Z, Z)$: "Ironing"

- The mapping $y \longrightarrow Z$ is always increasing $\Longrightarrow$ Existence of Cournot equ.
- Industry output higher, so tougher competition under pure Cournot.
- Industry viability is not a prominent issue under Cournot, so commitment model less suitable to capture industry failures.

Example

Model 1: RECE with $P(Z,S) = S(a - bZ)$

Price elasticity is independent of S.

$\pi(x,y,S) = xS\left[a - b(x+y)\right] - cx$

Then $Z_n(S) = \frac{n(aS-c)}{b(n+1)S} = S$

- There are

1. 3 equilibria (if $n > 3$ and $a^2 > 4bc$)

$$Z_n = 0 \text{ and } Z_n = \frac{an \pm \sqrt{a^2 n^2 - 4cbn(n+1)}}{2b(n+1)}$$

   Conditionally viable industry.

2. 1 equilibrium if $n < 3$: $Z_n = 0$ and industry is non-viable.

Model 2: Cournot with $P(Z, Z) = Z(a - bZ)$

FOC: $3bx^2 + 2(2by - a)x + by^2 - ay - c = 0$

Unique Cournot equ. with (if $a^2 > 4bc$)

$$Z_n = \frac{a(n+1) + \sqrt{a^2(n+1)^2 - 4cbn(n+2)}}{2b(n+2)}$$

Comparison

- Industry output higher under commitment model, so tougher competition
- Industry viability higher under commitment model : viability can be an issue only for usual high cost reasons, but not for expectations-based reasons.